Objective & Impact of Professor’s Research

Professor Assad Oberai’s research on neural networks covers a wide range of applications including solving inverse problems in physics, image processing for medical diagnosis, and exploring physics informed neural networks (PINNS). PINNS is a specific type of neural network in which a computer is “trained to solve supervised learning tasks while respecting any given laws of physics.” As a relatively new method of solving complex differential equations, there are still much to discover in PINNS and its unique potential. This research involving Ph.D. students Harisankar Ramasamy and Orazio Pinti specifically focused on PINNS’s ability to solve the Helmholtz Equation given differing values of wavenumber \( k \) and the implementation of the Fourier method.

Methods

**Helmholtz Equation**: commonly used to model the propagation of acoustic, electromagnetic, and quantum waves. \( d^2u/dx^2 - k^2u = 0, \ x \in [0, 1] \)

\( u(0) = 0 \)

\( u(1) = 1 \)

**Neutral Network**: type of machine learning in which a computer learns to perform a certain task by analyzing training data.

**Objective**: Observe effects of increasing \( k \) & \( \lambda_b \)

**Parameters**

- **Width**: 30
- **Depth**: 4
- **Optimiser**: function: \( \sin(x) \)

**Loss vs \( k \) and \( \lambda_b \)**

- **\( k \)**: \( 3.5\pi, 4.5\pi, \ldots 9.5\pi, 10.5\pi \)
- **\( \lambda_b \)**: \( 10, 100, 1k, 10k \)
- **Increase epochs until convergence**
- **\( N \)**: 50

\( N \) is the number of prediction points used to compute the loss.

**Loss Function**: \( L_{total} = L_{PDE} + \lambda_b L_{boundary} \)

**Objective**: Observe effects of Fourier method & increasing \( N \)

**Results**

**Figure 1**: Greater \( k \) value requires greater \# of epochs for convergence

- \( \lambda_b \) value becomes more significant with greater \( k \) value (requires greater \( \lambda_b \) for convergence)

**Figure 2**: Fourier’s method significantly reduces \# of epochs required

- Convergence occurs much faster (~150k vs <3k)

**Figure 3**: Greater \( N \) value lessens average error for greater \( k \) values

- For \( k \) values less than 10.5\( \pi \), the average error remains constant given values of \( N \)

Applications to STEM Coursework/Next Steps

While conducting research on the Helmholtz equation, I came across familiar topics from my calculus, statistics, and physics classes. It was interesting to see how materials from three different courses combined to create the framework for PINNS and its innerworkings. I also learned new skills and concepts such as PDEs, gradient descent, tensorflow, numpy, and python plotting. PINNS is still a relatively new type of machine learning, and its potential is not yet fully discovered. Thus, as part of my engineering journey, I would like to further explore neural networks in the future as well as continue developing new skills in coding and mathematics.

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